Sequences - Quant Study Notes for Competitive Exams

Here in this blog, we are going to explore **Sequences and Series** as we see many questions every year to be asked in the almost all exams and this is the easiest and most important concept which we should not leave during preparations. So, let's study sequence first.

What is the sequence?

A sequence is a set of numbers in which numbers occur in a definite order or governed by a rule. For a simple understanding when we are in an unknown colony to find a specific house number. We simply notice that house numbers are in order e.g. (61,62,63...), Yes, it is a sequence of house numbers.

Progressions

Sequences of special types are called progressions where the terms of the sequence follow a particular pattern. **Types of Progression:**

1. Arithmetic Progression
2. Geometric Progression
3. Harmonic Progression

We will discuss all three of them one-by-one.

Arithmetic Progression

A sequence a_1 , a_2 , a_3 ... a_{-n} where $a_2 - a_1 = a_3 - a_2 = a_n - a_{n-1}$, Where quantities increased or decreased continuously with a common quantity. E.g. 1, 3, 5, 7, 9, ... Here, Common difference = 2 & First term = 1

General term (nth term) of A.P.:

There is a rule binding all the terms of progression so we could tell more about the terms by following that rule By going though previous example we saw: ⇒ First term = 1 ⇒ Second term = 3 = 1 + 2 (one times 2) ⇒ Third term = 5 = 1 + 2 + 2 (two times 2) ⇒ Fourth term = 7 = 1 + 2 + 2 + 2 (three times 2) Similarly, nth term will be = 1 + 2 + 2 ... (n - 1 times 2)

So, nth term would be = first term + (n - 1) × difference Hence, $\mathbf{T_n = a + (n - 1) \times d}$ Where, T_n = nth term of A.P., a = First term of A.P., n = Number of terms in a progression, d = Difference between terms

E.g. Find the 17th term of -7, -2, 3, 8, 13, ... Sol: Here, n = 17, a = -7, d = -2 - (-7) = 5, 3 – (-2) = 5 So according to formula, ⇒ T_{17} = a + (n - 1) × d= -7 + (17 - 1) × 5 = -7 + 16 × 5 = 73 (Ans.)

Sum of n terms of A.P.: $S_n = n/2 \times [2a + (n - 1) \times d]$ We can write it as, $S_n = n/2 \times [a + a_n]$

E.g. Find the sum of 11 terms of -7, -2, 3, 8 ... Sol: We noticed that the terms are A.P. So by the sum would be $S_n = n/2 \times [2a + (n - 1) \times d]$ Where, n = 11, a = -7, d = 5 By substituting the values, S_n = 11/2 × [2 × (-7) + (11 - 1) × 5] = 11/2 × [-14 + 10 × 5] = 11/2 × [-14 + 50] = 11/2 × [36] = 11 × 18 = 198 (Ans.)

Special Point: When three quantities are in A.P. then the middle one is called the arithmetic mean of the other two terms. **A.M. = (a + b)/2**

E.g. If the sum of the first five terms of an A.P. is 75, then find the third term of the A.P. Sol: Let the terms of A.P. be a – 2d, a – d, a, a + d, a + 2d respectively. According to question sum would be ⇒ a – 2d + a – d + a + a + d + a + 2d ⇒ 5a = 75 (Given) ⇒ a = 15 (Ans.)

As we know the third term is 'a' so our answer would be 15.

TIP: If we have to let terms in A.P. then we should assume the middle term 'a' and the term before them as 'a - d', 'a - 2d'... and terms after them should be 'a + d, 'a + 2d' ... so we could cancel out terms easily.

Harmonic Progression

A sequence of quantities a_1 , a_2 , a_3 ... an is said to be H.P. when their reciprocals are in A.P. ⇒ $1/a_2 - 1/a_1 = 1/a_3 - 1/a_2 = 1/a_4 - 1/a_3$ For example a sequence of terms like 1, 1/2, 1/3, 1/4, ... There is no general formula for H.P. but we can solve these questions by inverting the terms and using properties of A.P.

Harmonic Mean: When three quantities are in H.P. The middle one is called the Harmonic Mean of the other two.

E.g. Find 10th term of H.P. 1, 1/3, 1/5, 1/7, 1/9, ... Sol: So by inverting the progression we saw an A.P. 1, 3, 5, 7, 9... It's 10th term would be, ⇒ T_{10} = a + (n - 1) × d = 1 + (10 - 1) × 2 = 1 + 9 × 2 = 19 By inverting 19 we got 1/19 so our answer is 1/19.

Geometric Progression

A sequence a_1 , a_2 , a_3 ... an is said to be in G.P. when $\mathbf{a_2 /a_1 = a_3 /a_2 = a_4 /a_3 ... an /a_{n-1} = r}$ Where a_1 , a_2 , a_3... an are non-zero numbers and 'r' is said to be a common ratio of the G.P.

E.g. 2, 4, 8, 16, 32... We saw in this example that as the first term is 2 then the next term would be 2 times of that. Similarly, the third term is twice the second term so it is a G.P.

General Term (nth term) of G.P.: The general term of a geometric progression is $\mathbf{T_n = a.r^{n-1}}$ Where, T_n = nth term, a = First term r = Common ratio, n = Number of terms

- If the common ratio (r) of G.P. is greater than 1 then the term strictly increases. So we can see, $a_n > a_{n-1} > ...a_4 > a_3 > a_2 > a_1$.
- Similarly when the common ratio (r) of G.P. is less than 1 then the terms strictly decrease. So we can see $a_n < a_{n-1} < ... a_4 < a_3 < a_2 < a_1$.
- When common ratio (r) = 1, then all the terms would be equal $a_n = a_{n-1} = ... a_4 = a_3 = a_2 = a_1$.

Sum of terms of G.P.:

When the common ratio > 1 $S_n = [a(r^n - 1)] / (r - 1)$ When the common ratio < 1 $S_n = [a(1 - r^n)] / (1 - r)$

When common ratio = 1 $S_n = n \times a$

Now we will try to solve some questions related to it.

E.g. Find 8th, 11th term of G.P. and find the sum of the first 11 terms. Sol: G.P. = 3, 6, 12, 24, 48,... In this sequence we saw that the difference of terms is not equal: 6 – 3 ≠ 12 – 6 But we can see another type of rule here as 6/3 = 12/6 = 24/12 = 48/24 = 2 Here the ratio of terms is equal; it is a G.P. From the nth term formula ⇒ $T_8 = a.r^{8-1-}$ ⇒$T_8 = 3 \times 2^7 = 3 \times 128 = 384$ (Ans.) Similarly, $T_{11} = a.r^{11-1}$ ⇒$T_1 1 = 3 \times 2^{10} = 3 \times 1024 = 3072$ (Ans.) From the formula of the sum of 'n' terms Sum of first 11 terms would be = $[a(r^n - 1)]/(r - 1)$ (r > 1) $S_{11} = [3 \times (2^{11} - 1)]/(2 - 1) = [3 \times (2048 - 1)]/1 = 6141$ (Ans.)

E.g. 256, 192, 144, (?) Sol: Here the difference is not equal so let's check the common ratio ⇒192/256 = 144/192 = 3/4 OHH so it's a decreasing G.P. with common ratio 3/4 So, fourth term would be $T_4 = a.r^{4-1}$ ⇒$T_4 = 256 \times (3/4)^3 = 256 \times 27/64$ = 108 (Ans.)

Special point: If the three quantities are in G.P., the middle one is called the geometric mean (G.M.) of the other two quantities. If a, m, b are the terms of G.P. $m = \sqrt{ab}$ OR $m^2 = a \times b$

So, it will be all for sequences. You should try to solve some of the questions regarding this topic.