Today we are going to learn about Percentages. We saw 2-3 questions are asked every year from percentages. Let's first learn what we mean by percentage.

The percentage is derived from the Latin word "percentum" which means "per hundred" and it is denoted by %. A fraction with denominator 100 is called percentage. Similarly, a fraction with denominator 10 is called a decimal.

Fraction into percentage

To convert a fraction 'a/b' in percentage, multiply it with 100 and put a '%' sign.

E.g. Convert 4 / 5 into a percentage.

Sol: We have to multiply this fraction to 100 first $\Rightarrow$ 4 / 5 × 100 = 400 / 5 = 80 Now we have to put a '%' sign at the end ☐ 80% (Ans.)

There are some frequently occurring fractions so we should remember them. Let us learn those important fractions.

1/1 = 100%	1/2 = 50%	1/3 = 33.33%	1/4 = 25%	1/5 = 20%
1/6 = 16.66%	1/7 = 14.28%	1/8 = 12.5%	1/9 = 11.11%	1/10 = 10%
1/11 = 9.09%	1/12 = 8.33%	1/13 = 7.69%	1/14 = 7.14%	1/15 = 6.66%
1/16 = 6.25%	1/17 = 5.88%	1/18 = 5.55%	1/19 = 5.26%	1/20 = 5%

Percentage into fraction

To convert the percentage into fraction, divide the number by 100 and remove the % sign.

E.g. Convert 9.09% into fraction form.

Sol: First, we have to divide this by 100 $\Rightarrow$ 9.09 / 100 = (100 / 11%) / 100 (As we know 9.09 can be written as 100 / 11) $\Rightarrow$ 1 / 11% Now remove the % sign $\Rightarrow$ 1 / 11 (Ans.)

E.g. A student scored 85% marks. The total marks were 400. How much did he score?

Sol: As his marks are 85% of total marks and total marks are 400 $\Rightarrow$ His score = 85% of 400 $\Rightarrow$ His score = (85 / 100) × 400 $\Rightarrow$ His score = (85) × 4 = 340 (Ans.)

Effect of Percentage Change on Any Quantity:

Let a number 'p' be increased by 'x%' then new quantity will be $\Rightarrow \{(100 + x) / 100\} \times p$ Similarly, when 'p' decreases by 'x%' then new quantity will be $\Rightarrow \{(100 - x) / 100\} \times p$

E.g. The present salary of Amit is 3000. His salary will be increased by 15% next year, find his increased salary.

Sol: His salary is increased by 15% so here x is 15 According to formula, $\{(100 + x) / 100\} \times p$ By substituting the values $\Rightarrow \{(100 + 15) / 100\} \times 3000 \Rightarrow \{115 / 100\} \times 3000 = 115 \times 30 = 3450$ (Ans.)

Here, we saw there could be an increase or decrease in quantity by some percentage.

Succesive increase or decrease of percentage

What if the quantity (p) is changed in two steps like in the first step it increases or decreases by 'x%' and in second step it increases or decreases by 'y%' then total percentage change would be $\Rightarrow [\{(100 \pm x)\} / 100] \times [\{(100 \pm y)\} / 100] \times p$

The sign will be positive when value increases and negative when value decreases. Let's take an example to understand this.

E.g. If the length of the rectangle increases by 30% and the breadth of the rectangle decreases by 12%. Find the overall change in the area of a rectangle.

Sol: Let the area of the rectangle be 'a' square unit, and we know that values of x and y are 30 and 12 respectively.

According to our formula $\Rightarrow [\{(100 \pm x)\} / 100] \times [\{(100 \pm y)\} / 100] \times p$. By substituting all values, $[\{(100 + 30)\} / 100] \times [\{(100 -12)\} / 100] \times a \Rightarrow [130 / 100] \times [88 / 100] \times a \Rightarrow [13 / 10] \times [22 / 25] \times a \Rightarrow (286 / 250) \times a = 1.144x$ Our difference would be $1.144x - x = 0.144$ OR 14.4% (Ans.)
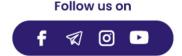
For this type of question, we don't have to opt these long conventional methods, we have to just spell a magic formula, Total percent change = x + y + xy/100. Where 'x' and 'y' are percentage changes on quantity. Signs of 'x' and 'y' would depend on fluctuation in quantity.

E.g. If the length of the rectangle increases by 30% and the breadth of the rectangle decreases by 12%. Find the overall change in the area of the rectangle.

Sol: Here x = +30 and y = -12 By putting values, $x + y + xy/100 = 30 - 12 - 30 \times 12/100 = 18 - 36/10 = 18 - 3.6 = 14.4\%$ (Ans.)

E.g. If the length of the rectangle increases by 30% and the area doesn't change. Find the change in the breadth of the rectangle.

Sol: So here the question is twisted little bit, we have given total change as 0% and x = 30 and we have to find 'y' Total change = $x + y + xy/100 \Rightarrow 0 = 30 + y + (30 \times y)/100 \Rightarrow -30 = y + (3y) / 10 \Rightarrow -30 = (1 + 0.3) \times y \Rightarrow y = -30 / 1.3 = -23.07\%$ (Ans.)

We saw a decrease of 23.07% When one factor of a product increases by 'p%' then the other factor will decrease by {p / (100 + p)} × 100%, similarly when one factor of a product decreases by 'p%' then the other factor will increases by {p / (100 - p)} × 100% so the total change would be zero in both cases.

E.g. If the price of coffee is increased by 10% then by how much percentage a housewife decreases the consumption, to have no extra expenditure.

Sol: We know that Expenditure = Consumption × Price By substituting the values, {p / (100 + p)} × 100% ⇒{10 / (100 + 10)} × 100% ⇒ {10 / 110} × 100% = (1 / 11) × 100% = 9.09% (Ans.)

E.g. If the numerator of a fraction is increased by 200% and the denominator of the fraction is increased by 150%, the resultant fraction is 9/25, find the original fraction.

Sol: Let the numerator be 'x' and denominator be 'y' resultant would be: {x + 2x} / {y + (3 / 2 × y)} which is equivalent to 9/25 ⇒3x / 2.5y = 9 / 25 ⇒x / y = 3/10 (Ans.)

E.g. If the income of Shyam is 50% more than Ram, then how much percent of Ram's income is less than Shyam?

Sol: Ram's income would be less by, ⇒{p / (100 + p)} × 100% = {50 / (100 + 50)} × 100% = {50 / 150} × 100% = {1 / 3} × 100% = 33.33% (Ans.)

To solve all the mathematics problems, it is crucial to master the concept of percentages, which is a fundamental component of the quantitative section. To excel in this area, one must be able to apply the relevant concepts with accuracy and speed. So, keep practicing and stay tuned for more.