HCF and LCM - Quant Study Notes for Competitive Exams

Hi guys, we will presently learn about HCF and LCM. Let's get started then, firstly we will talk about what is HCF and LCM.

HCF: Highest Common Factor (HCF) is the largest factor of two or more given numbers which can divide all of these numbers. It is also called Greatest Common Divisor (GCD).

LCM: Lowest Common Multiple (LCM) of two or more numbers is the least number which is divisible by each of these numbers (i.e. leaves no remainder). [Note: Product of two numbers = Their LCM × Their HCF]

E.g. Find the HCF and LCM of 288, 432 and 768.

Sol: 288 = 25 × 32 ⇒432 = 24 × 33 ⇒768 = 28 × 3 So, HCF would be = 24 × 3 = 48 And, LCM would be = 28 × 33 = 6912.

Now, here are some interesting ways to find HCF of two or more numbers.

1) Long Division method (Best applicable for two numbers): Take two numbers. Divide the greater number by the smaller number, then divide the divisor by the remainder, divide the divisor of this division by the next remainder and so on until the remainder is 0. The last divisor is the HCF of the two numbers taken.

E.g. Find the HCF of 1363 and 1457.

Sol: 1457/1363

HCF (1367, 1457) = 47.

2) Difference method (Best applicable for numbers which are close to each other): Let us take the numbers like 3800, 3600, 3672. First we have to find the minimum difference between numbers (in this case, it is 3672 – 3600 = 72). Then we have to check whether 72 or which factor (highest) of 72 is also a factor of the third number (i.e. 3800) so here, 72 = 23 × 32 . Since 3 cannot divide 3800, no multiple of 3 can divide 3800. So, we are left with 23 , we can see that 3800 can be divided by 8 but not by any factor of 72, bigger than 8. So, 8 is the HCF of these numbers. Let us take another example, find the HCF of 2704, 2700, 1586. Here, the minimum difference is 4 (i.e. 2704 – 2700). Since, 4 cannot divide 1586 but its factor 2 does. And 2 is the second biggest factor of 4. So, 2 is the HCF of these numbers.

3) Factor method (Best applicable for numbers in which at least one number is small enough to calculate the factors): Let us take numbers like 108, 288 and 360. Here the smallest number is 108 but it does not divide 288 and 360 both. Now its second biggest factor is 54, but it also cannot divide 288 and 360 both. Now its third biggest factor is 36, yes it can divide both 288 and 360. So, 36 is the HCF of given numbers.

HCF and LCM of fractions

LCM = (LCM of Numerators)/(HCF of Denominators)

HCF = (HCF of Numerators)/(LCM of Denominators)

It means, if there are numbers like a/b, c/d and e/f. Then, LCM (a/b, c/d, e/f) = [LCM (a, c, e)]/[HCF (b, d, f)] HCF (a/b, c/d, e/f) = [HCF (a, c, e)]/[LCM (b, d, f)]

[Note: Fractions should be in reduced forms i.e. if the fraction is like x/y, x and y must be co-prime numbers]

E.g. Find the LCM of 3/7, 5/9, 4/10 and 8/9.

Sol: Here, most of the students make mistakes. They directly take the LCM of (3, 7, 4 and 8). But here, we have to first simplify the fraction. So, 4/10 will become 2/5. Now, LCM = (LCM of 3, 5, 2, 8)/(HCF of 7, 9, 5, 9) = 120.

Now, let us just look at some properties and tricks of HCF and LCM:

Any number which when divided by p, q or r leaving the same remainder s in each case, will be the form of k (LCM of p, q, r) + s where k = 0, 1, 2, … . If k is 0, then we get the smallest such number.

E.g. Find the smallest number which when divided by 4, 11 or 13 leaves a remainder 1 and is greater than the remainder.

Sol: Required number = LCM (4, 11, 13) + 1 = 573.

Any number which when divided by p, q or r leaving respective remainders of s, t and u where (p – s) = (q – t) = (r – s) = a (say), will be of the form k (LCM of p, q and r) – a. The smallest such number will be obtained by substituting k = 1.

E.g. Find the smallest numbers which when divided by 8 and 12 leave remainders of 6 and 10 respectively. Sol: Required number = LCM (8, 12) – 2 = 22.

The largest number with which the numbers p, q or r are divided gives remainders of s, t and u respectively will be the HCF of the three numbers (p – s), (q – t) and (r – u).

E.g. Find the largest number which leaves the remainders of 2 and 3 when it divides 89 and 148 respectively.

Sol: Largest number = HCF (89 – 2, 148 – 3) = HCF (87, 145) = 29.

If there were questions like, Find the largest number with which if we divide the numbers p, q and r, the remainder are the same. Then the required number is HCF of (p – q) and (p – r) = HCF of (p – q) and (q – r) = HCF of (q – r) and (p – r). It is happening because the numbers have the same remainder, so if we subtract the remainder from the numbers, the numbers will become the multiples of that number. It means, the difference between those multiples is equal to the difference between p, q and r. And since the numbers are multiples, the difference between them is also a multiple of that number.

E.g. Find the largest number which divides 444, 804 and 1344 leaving the same remainder in each case. Sol: Largest number = HCF (804 – 444, 1344 – 804) = HCF (360, 540) = 180.

Now, before binding up this blog, let us look at a good and interesting example.

E.g. Mahesh distributed all the marbles with him equally among 8 children and found that 5 marbles were left. Had he distributed the marbles equally among 12 or 18 children, he would have still had 5 marbles left with him. If the number of marbles Mahesh distributed was less than 200, how many marbles did he initially have?

Sol: Try this question yourself before looking at my solution. If we remember the formula, this question will not take more than 2 minutes. The number of marbles Mahesh had = k × [LCM (8, 12, 18)] + 5 (where k is any natural number) Now, LCM (8, 12, 18) = 72. So, the number of marbles Mahesh had = 72k + 5 Since, the number of marbles he had is less than 200 i.e. 72k + 5 < 200 ⇒k = 1 or 2 So, the number of marbles Mahesh had could be 72 + 5 or 72 × 2 + 5 i.e. 77 or 149.

Hope you understood the whole topic and further we will be dealing with a important topic. Stay connected with us.