Factors and multiples- Quant Study Notes for Competitive Exams

Hi guys, presently we are here to talk about Factors and multiples. Let's get started. Firstly, let's just know the meaning of factors and multiples.

Factors: Factors of a number are the numbers by which when the number is divided, the remainder will be 0.

Multiples: Multiples of a number are the numbers which are if divided by the number, the remainder will be 0. E.g. 8 has the factors 1, 2, 4 and 8 and has the multiples 8, 16, 24, etc.

Now, let's talk about factors.

Any number greater than 1 can be written as the product of prime factors. Let's generalize this term then we will explore this topic.

Let N is a composite number. So, by above statement, N can be written as, **N = ap × bq × cr × …** (Where a, b, c are distinct prime factors of N and p, q, r are integers) So, the general formula to calculate the number of factors of N is: Total number of factors of N = (p + 1) (q + 1) (r + 1) …

E.g. Find the total number of factors of 756.

Sol: 756 can be written as, $756 = 2^2 \times 3^3 \times 7$ So, the total number of factors of 756 = (2 + 1) (3 + 1) (1 + 1) = 3 × 4 × 2 = 24.

E.g. How many factors of 756 are multiple of 12?

Sol: 756 = 22 × 33 × 7 and 12 = 22 × 3 ⇒756 = 22 × 31 [32 × 71] Since we have to find out the multiples of 12 in it. So, this [32 × 71] is important for us. No. of factors = (2 + 1) (1 + 1) = 3 + 2 = 6.

Co-Prime Numbers: Co-Prime numbers are the two numbers which have no common factor other than 1. e.g. 24 and 25 are co-prime numbers because 24 has factors 1, 2, 3, 4, 6, 8, 12, 24 and 25 has factors 1, 5, 25. No factor is common other than 1.

Now let us take a look at some interesting points:

1. The product of two consecutive integers is always even. Because one of them is even and one of them is odd, so the product will always be even.
2. The product of three consecutive integers is always divisible by 6. Because one of them is always even and one of them is always divisible by 3.
3. Like that, the product of four consecutive integers is always divisible by 24.
4. The product of n consecutive integers will always be divisible by n.
5. Total one digit numbers are 9. Total two digit numbers are 90 (from 10 to 99). So, they have 90 × 2 = 180 digits. Total three digit numbers are 900 (from 100 to 999). So, they have 900 × 3 = 2700 digits. 35 Total four digit numbers are 9000 (from 1000 to 9999). So, they have 9000 × 4 = 36000 digits. And so on.

E.g. Divya typed the first n natural numbers on a keyboard without any spaces. If she had to press keys 1692 times, find n.

Sol: 1 digit numbers have 9 digits. 2 digit numbers are 90 and have 180 digits. So, 1692 – (180 + 9) = 1503. Since 3 digit numbers are 900 and have 2700 digits but we left only 1503 digits, it means the number is a 3 digit number. Now, 1503/3 = 501, means it is the 501th 3 digit number. So, the number is, 501 + 90 + 9 = 600.

Let us look at two examples and if you are now confident about this section, just try these questions by yourself before looking at my solution. It would be fun.

E.g. How many natural numbers divide 35^{999} but do not divide 35^{998}?

Sol: It wants us to find the factors of 35^{999} which are not the factors of 35^{998}. In simple words, (No. of factors of 35^{999}) – (No. of factors of 35^{998}), Because these are the only factors which are greater than 35^{998} and, so that, they do not divide 35^{998} . Now, $35^{999} = (5 \times 7)^{999} = 5^{999} \times 7^{999}$ ⇒No. of factors = (999 + 1) (999 + 1) = 1000 × 1000 = 1000^2 Also, $35^{998} = (5 \times 7)^{998} = 5^{998} \times 7^{998}$ ⇒No. of factors = (998 + 1) (998 + 1) = 999 × 999 = 999^2 ⇒required answer = 10002 – 9992 = (1000 + 999) (1000 – 999) = 1999 × 1 = 1999.

E.g. In an NGO, there are 70 workers. All the workers visited an old age home having 70 senior citizens. The first worker donated Rs. 1000 to each senior citizen. The second worker donated Rs. 1000 to every second senior citizen starting from the second senior citizen. The third worker donated Rs. 1000 to every third senior citizen starting from the third senior citizen and so on. How many senior citizens received donations from an odd number of workers?

Sol: Read the question twice and thrice and try to understand what this question wants to say. The first worker gives rupees to all 70 citizens. But the 2^{nd} worker gives only to 2^{nd}, 4^{th}, 6^{th} , 8^{th}, etc. citizens. Same thing is done by 3^{rd}, he gives rupees to only 3^{rd}, 6^{th}, 9^{th}, 12^{th}, etc. citizens. Same for 4^{th}, 5^{th}, 6^{th} workers till 70^{th}. It means, a worker whose number is x is giving money to all the citizens whose number is a multiple of x. And hence, the citizen, whose number is y, is receiving the donations from all the workers whose number is a factor of y. i.e. is receiving as many donations as the number of factors of y. So, we just have to find out, from 1 to 70, how many numbers have odd numbers of factors. And from the previous discussion, we know that only perfect squares have an odd number of factors. And from 1 to 70, there are 8 perfect squares, which are 1, 4, 9, 16, 25, 36, 49 and 64. So, the answer is, only 8 senior citizens received donations from an odd number of worker.

Hope all your doubts have been clear after going through the blog. Stay connected for more.